PAPER

Digital Multiple Notch Filter Design with Nelder-Mead Simplex Method

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SUMMARY Multiple notch filters are used to suppress narrow-band or sinusoidal interferences in digital signals. In this paper, we propose a novel optimization design technique of an infinite impulse response (IIR) multiple notch filter. It is based on the Nelder-Mead simplex method. Firstly, the system function of the desired notch filter is constructed to form the objective function of the optimization technique. Secondly, the design parameters of the desired notch filter are optimized by Nelder-Mead simplex method. A weight function is also introduced to improve amplitude response of the notch filter. Thirdly, the convergence and amplitude response of the proposed technique are compared with other Nelder-Mead based design methods and the cascade-based design method. Finally, the practicability of the proposed notch filter design technique is demonstrated by some practical applications.

key words: *multiple notch filters, digital filter design, narrow-band interference suppression, Nelder-Mead simplex method*

1. Introduction

Multiple notch filters are used to eliminate or suppress some undesired sinusoidal or narrow-band interferences in a digital signal while keep other components unchanged. The applications of notch filters range from power-line noise removal, speech signal estimation to biomedical signal processing suppression [1]–[5]. Generally speaking, notch filters can be classified into two different types: finite-impulse response (FIR) notch filters and infinite-impulse response (IIR) notch filters. The former possess better characteristics of linearity and stability, while the latter have narrower stop-band and higher quality factor under the same orders. There are two kinds of IIR notch filters: single notch filters and multiple notch filters. The former are used to eliminate the single narrow-band component, which can be directly designed by a second-order section. The latter are used to remove multiple interference components simultaneously.

In IIR notch filters, there are adaptive and non-adaptive notch filters. This paper focuses on the problem of non-adaptive notch filter, which are designed with specified notch frequencies. For IIR multiple notch filters, there are three common design methods: cascade method [6], all-pass filter-based method [7]–[10] and optimal design method [11]–[13]. In cascade method, a multiple notch filter is realized by cascading several second-order sections of single notch filters. However, it may yield an unsatisfied amplitude response. In all-pass filter-based method, an all-pass filter is used to construct the notch filter. A classical design was proposed in [7], which is widely used because of the simplicity of solving the linear equations. But the design equations may result in an ill-condition due to the tangent operations. The optimal design method places the poles at optimal positions to get a good amplitude response. A typical optimal design based on quadratic programming was proposed by Tseng and Pei [11]. It introduced Rouché's theorem to ensure the stability. However, in some situations, poles may not be placed on the specified circles.

Generally speaking, in the optimal methods of notch filter design, the parameter optimization is often complicated. The Nelder-Mead (NM) simplex method was first proposed by J.A. Nelder and R. Mead [14]. Afterwards, many researchers put forward some improved NM-methods [15]–[18], which have provided some effective iteration methods. Because the NM-method does not require gradient computation, it is widely used in some complex engineering problems, such as training neural network problem, structural inverse, composite laminate design and economic dispatch problem [15], [19]–[21]. In this paper, the Nelder-Mead simplex method is introduced to reduce the computation complexity of multiple notch filter design.

The rest of the paper is organized as follows. In Sect. 2, the system function of digital IIR multiple notch filter is constructed and the classical Nelder-Mead simplex method is introduced. Then, the Nelder-Mead method is used to improve design performance. In Sect. 3, a set of experiments are carried out to validate the effectiveness and practicability of the proposed method. Finally, the conclusion of this paper is drawn in Sect. 4.

2. IIR Notch Filter Design by Nelder-Mead Method

2.1 System Function of Multiple Notch Filter

The ideal frequency response of a multiple notch filter with N notch frequencies is described by

$$H_d(e^{j\omega}) = \begin{cases} 0, & \omega = \omega_i \\ 1, & \text{otherwise} \end{cases} \tag{1}$$

where ω_i is the i-th notch frequency, for $i = 1, 2, \cdots, N$.

In order to approximate the frequency response of the ideal notch filter, the system function of the desired filter can be constructed by N second-order sections as follows

Manuscript received July 1, 2016.
Manuscript revised September 13, 2016.

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DOI: 10.1587/transfun.E100.A.259

$$H(z) = K\frac{\prod_{i=1}^{N}(1-e^{j\omega_i}z^{-1})(1-e^{-j\omega_i}z^{-1})}{\prod_{i=1}^{N}(1-r_ie^{j\hat{\omega}_i}z^{-1})(1-r_ie^{-j\hat{\omega}_i}z^{-1})} = K\frac{\prod_{i=1}^{N}(1-2\cos(\omega_i)z^{-1}+z^{-2})}{\prod_{i=1}^{N}(1-2r_i\cos(\hat{\omega}_i)z^{-1}+r_i^2z^{-2})} \quad (2)$$

where K is the constant coefficient used to normalize amplitude response, ω_i is the i-th notch frequency, $\hat{\omega}_i$ and r_i are the polar angle and radius of the i-th pair poles, respectively. From (2), the zeros $e^{\pm j\omega_i}$ are placed on the unit circle but the poles $r_ie^{\pm j\hat{\omega}_i}$ are located inside to guarantee the system stability, i.e. $0 < r_i < 1$, for $i = 1, 2, \cdots, N$.

Let $z = e^{j\omega}$, the frequency response of $H(z)$ can be obtained from (2) and represented as

$$H(e^{j\omega}) = K\frac{\prod_{i=1}^{N}(1-2\cos(\omega_i)e^{-j\omega}+e^{-j2\omega})}{\prod_{i=1}^{N}(1-2r_i\cos(\hat{\omega}_i)e^{-j\omega}+r_i^2e^{-j2\omega})} \quad (3)$$

In cascade design method, the polar angles are set the same as the corresponding notch frequencies, i.e. $\hat{\omega}_i = \omega_i$, for $i = 1, 2, \cdots, N$. That is only effective under narrow bandwidth conditions. If the gaps between notch frequencies are narrow, the amplitude response will be corrupted. The solving of $\hat{\omega}_1, \hat{\omega}_2, \cdots, \hat{\omega}_N$ is a multi-variable optimization problem. An optimal design method was proposed to decrease amplitude response distortions, but the problem of unmatched polar radius occurs [11].

In this paper, we propose a new design technique to improve the performance of multiple notch filter. The optimization problem of solving polar angles will be implemented by the Nelder-Mead simplex method.

2.2 Nelder-Mead Simplex Method

The Nelder-Mead simplex method (NM-method) is a local search method [14]. It is suitable to solve a multi-variable optimization problem, especially an unconstrained problem

$$\min f(\mathbf{x}) \quad (4)$$

where $\mathbf{x} = [x_1, x_2, \cdots, x_N]^{\mathrm{T}}$ is a vector to be optimized and $f : \mathbb{R}^N \to \mathbb{R}$ is an unimodal function.

In a NM-method, the simplex is a geometric figure in N dimensions. The geometric figure can be constituted by M vertexes $\mathbf{x}_{(1)}, \mathbf{x}_{(2)}, \cdots, \mathbf{x}_{(M)}$, where $M \geqslant N + 1$. Their geometric expressions are shown in Fig. 1. The flowchart of the NM-method is shown in Fig. 2, which is mainly constituted by initialization and iteration. In iteration, four key updates are termed Reflection, Expansion, Contraction and Shrink, where α, β, γ and δ are parameters in NM-method. The major steps of NM-method are explained as follows:

- **Input.** Input the objective function $f(\mathbf{x})$ and maximum iteration number L.
- **Initialization.** Generate the initial $\mathbf{x}_{(1)}, \mathbf{x}_{(2)}, \cdots, \mathbf{x}_{(M)}$ and set the iteration number $k = 1$.
- **Sort.** Compute the value $f_i = f(\mathbf{x}_{(i)})$ and sort vertexes $\mathbf{x}_{(1)}, \mathbf{x}_{(2)}, \cdots, \mathbf{x}_{(M)}$ to meet: $f_1 \leqslant f_2 \leqslant \cdots \leqslant f_M$.
- **Centroid.** Calculate the centroid point $\mathbf{x}_{\mathrm{o}}$ of the *best*

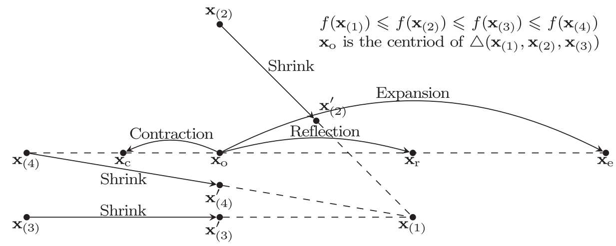


Fig. 1 Geometric expression of the NM-method when $M = 4$.

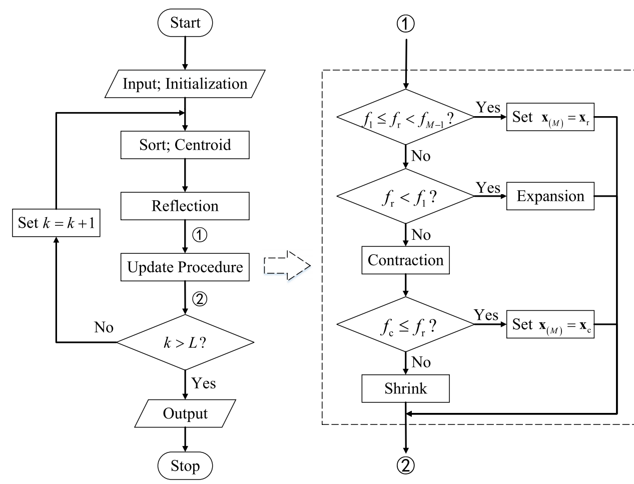


Fig. 2 Flowchart of the NM-method.

$M - 1$ vertexes:

$$\mathbf{x}_{\mathrm{o}} = \frac{1}{M-1}\sum_{i=1}^{M-1}\mathbf{x}_{(i)}$$

- **Reflection.** Calculate the reflection point $\mathbf{x}_{\mathrm{r}}$:

$$\mathbf{x}_{\mathrm{r}} = \mathbf{x}_{\mathrm{o}} + \alpha\cdot(\mathbf{x}_{\mathrm{o}} - \mathbf{x}_{(M)}), \quad f_{\mathrm{r}} = f(\mathbf{x}_{\mathrm{r}})$$

- **Expansion.** Calculate the expansion point $\mathbf{x}_{\mathrm{e}}$ and update $\mathbf{x}_{(M)}$:

$$\mathbf{x}_{\mathrm{e}} = \mathbf{x}_{\mathrm{o}} + \beta\cdot(\mathbf{x}_{\mathrm{r}} - \mathbf{x}_{\mathrm{o}}), \quad f_{\mathrm{e}} = f(\mathbf{x}_{\mathrm{e}})$$
$$\mathbf{x}_{(M)} = \begin{cases}\mathbf{x}_{\mathrm{e}}, & f_{\mathrm{e}} \leqslant f_{\mathrm{r}}\\ \mathbf{x}_{\mathrm{r}}, & \text{otherwise}\end{cases}$$

- **Contraction.** Calculate the contraction point $\mathbf{x}_{\mathrm{c}}$:

$$\mathbf{x}_{\mathrm{c}} = \begin{cases}\mathbf{x}_{\mathrm{o}} + \gamma\cdot(\mathbf{x}_{\mathrm{r}} - \mathbf{x}_{\mathrm{o}}), & f_{M-1} \leqslant f_{\mathrm{r}} < f_M\\ \mathbf{x}_{\mathrm{o}} - \gamma\cdot(\mathbf{x}_{\mathrm{r}} - \mathbf{x}_{\mathrm{o}}), & f_{\mathrm{r}} \geqslant f_M\end{cases}$$
$$f_{\mathrm{c}} = f(\mathbf{x}_{\mathrm{c}})$$

- **Shrink.** Update the *worst* $M - 1$ vertexes:

$$\mathbf{x}_{(i)} = \mathbf{x}_{(1)} + \delta\cdot(\mathbf{x}_{(i)} - \mathbf{x}_{(1)}), \text{ for } i = 2, 3, \cdots, M.$$

- **Output.** Output the current $\mathbf{x}_{(1)}$, which is the *best* vertex on current situation.

In above steps, four parameters are typically set as:

$$\alpha = 1, \quad \beta = 2, \quad \gamma = 0.5, \quad \delta = 0.5 \quad (5)$$

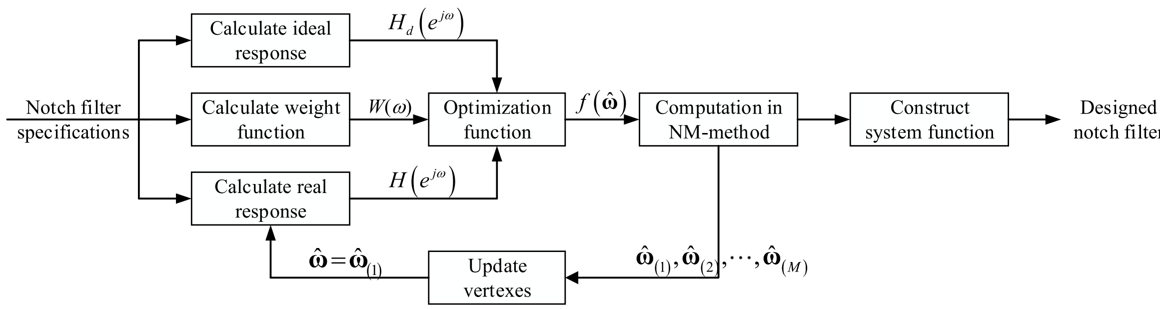


Fig. 3 Block diagram of the proposed multiple notch filter design.

Many researchers have studied the influences of those parameters and provided other setting values [16]–[18]. Those representative values used in this paper for notch filter design will be listed in Sect. 3.

2.3 Proposed Notch Filter Design Technique

Suppose the desired notch filter has N notch frequencies $\omega_1, \omega_2, \cdots, \omega_N$ and N bandwidths $\Delta\omega_1, \Delta\omega_2, \cdots, \Delta\omega_N$. The relationship between polar radius r_i and bandwidth $\Delta\omega_i$ is described as follows [13]

$$r_i = \sqrt{\frac{1 - \sin(\Delta\omega_i)}{\cos(\Delta\omega_i)}}, \quad \text{for } i = 1, 2, \cdots, N. \tag{6}$$

It is used to calculate the polar radius of i-th pair poles in (3), from i-th notch bandwidth.

To design a multiple notch filter using NM-method, the following objective function will be minimized in the proposed design technique

$$\min f(\hat{\omega}) = \int_0^\pi W(\omega) \left| H_d(e^{j\omega}) - H(e^{j\omega}) \right|^2 d\omega \tag{7}$$

where $W(\omega)$ is a weight function to improve the design performance, $H_d(e^{j\omega})$ and $H(e^{j\omega})$ are the ideal and real frequency responses expressed by (1) and (3), respectively. From (3) and (6), the polar angle $\hat{\omega} = [\hat{\omega}_1, \hat{\omega}_2, \cdots, \hat{\omega}_N]^{\mathrm{T}}$, to be optimized by NM-method, is closely related to notch frequencies and bandwidths.

The main idea of (7) is to minimize the weighted amplitude response error between the ideal and designed notch filters. According to our experience, $W(\omega)$ can be set as

$$W(\omega) = \begin{cases} 2, & \omega \in (\omega_i - \Delta\omega_i/2, \omega_i + \Delta\omega_i/2) \\ 1, & \text{otherwise} \end{cases} \tag{8}$$

It emphasizes the significance of the amplitude responses in notch regions and improves the amplitude response of the designed notch filter. The effectiveness of (8) will be demonstrated in Sect. 3.2.

According to [11], the objective function with same format as (7) was used to optimize amplitude response through quadratic programming, which is a special convex optimization. Therefore, solving of (7) can be considered as an unimodal optimization problem. So it can be solved effectively by Nelder-Mead simplex method.

Algorithm 1 Multiple Notch Filter Design Based on NM-method

Input: notch frequencies ω_i and notch bandwidth $\Delta\omega_i$ (for $i = 1, 2, \cdots, N$), vertexes number M and maximum iteration number L.
Calculate polar radius $r_1, r_2, \cdots, r_N$ via (6).
Compute $H(e^{j\omega})$ and $W(\omega)$ by (3) and (8), respectively. Afterwards, the objective function $f(\hat{\omega})$ is expressed via (7).
Randomly initialize M vertexes. For m-th vertex $\hat{\omega}_{(m)}$, make all of the elements $\hat{\omega}_{mn} \in (\omega_n - \Delta\omega_n/2, \omega_n + \Delta\omega_n/2)$, for $n = 1, 2, \cdots, N$ and $m = 1, 2, \cdots, M$.
Set $k = 1$.
while $k \leqslant L$ **do**
    Carry out **Sort** and **Centroid**, which are described in Sect. 2.2.
    Perform **Reflection**, **Expansion**, **Contraction** and **Shrink** operations, according to the flowchart of Fig. 2.
    Set $k = k + 1$.
end while
Output: polar angles $\hat{\omega}_{(1)} = [\hat{\omega}_{11}, \hat{\omega}_{12}, \cdots, \hat{\omega}_{1N}]^{\mathrm{T}}$. Note that $\hat{\omega}_{(1)}$ is the best vertex.
Complete the designed multiple notch filter via (2).

The block diagram of the NM-method based notch filter design technique is shown in Fig. 3. The m-th vertex in iteration is represented as $\hat{\omega}_{(m)} = [\hat{\omega}_{m1}, \hat{\omega}_{m2}, \cdots, \hat{\omega}_{mN}]^{\mathrm{T}}$, for $m = 1, 2, \cdots, M$. The realization procedure of the proposed design technique is summarized in Algorithm 1.

Different from other design techniques, the proposed one uses the NM-method to optimize the polar angles. Moreover, the weight function $W(\omega)$ is also introduced to improve the amplitude response of the designed filter. The advantages of Algorithm 1 will be validated by a set of experiments in the next section.

3. Experiments and Result Analysis

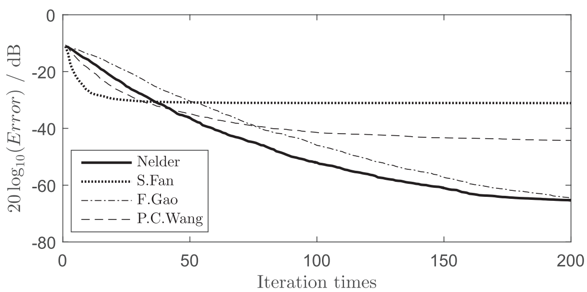
3.1 Convergence Comparisons among NM-Methods

In order to demonstrate the effectiveness of the proposed notch filter design method (shown in Algorithm 1), the convergences of different NM-methods are compared. The design specifications of notch filter (including notch frequencies and bandwidths) are listed in Table 1. For all NM-methods, the total vertexes number $M = 10$, the maximum iteration number $L = 200$.

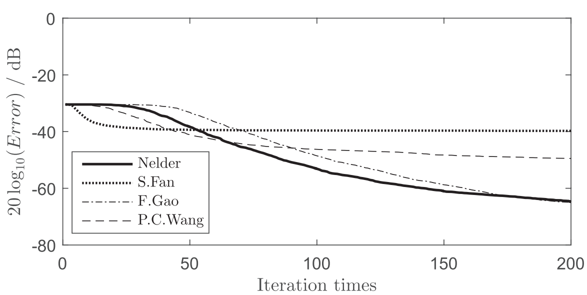
In general, the optimizing variables are often initialized randomly in the NM-methods. But in multiple notch filter design, the polar angles are generally closed to notch fre-

Table 1 Design specifications of multiple notch filter.

i	1	2	3	4
ω_i	0.1π	0.2π	0.4π	0.7π
$\Delta\omega_i$	0.06π	0.06π	0.06π	0.06π
r_i	0.9095	0.9095	0.9095	0.9095



(a) Initialized with random values.



(b) Initialized with specified values.

Fig. 4 Average design error of the NM-based notch filter with different parameters setting methods.

Table 2 Four types of parameters setting methods.

	Nelder	S.Fan [18]	F.Gao [17]	P.C.Wang [16]
α	**1.00**	1.50	1.00	$\alpha \sim N(1.29, 0.47^2)$
β	**2.00**	2.75	$1 + 2/N$	$\beta \sim N(2.29, 0.44^2)$
γ	**0.50**	0.75	$0.75 - 1/(2N)$	$\gamma \sim N(0.47, 0.17^2)$
δ	**0.50**	0.50	$1 - 1/N$	$\delta \sim N(0.57, 0.19^2)$

quencies. Hence, the best vertex $\hat{\omega}_{(1)}$ can be initialized as the notch frequency vector $[\omega_1, \omega_2, \cdots, \omega_N]^{\mathrm{T}}$.

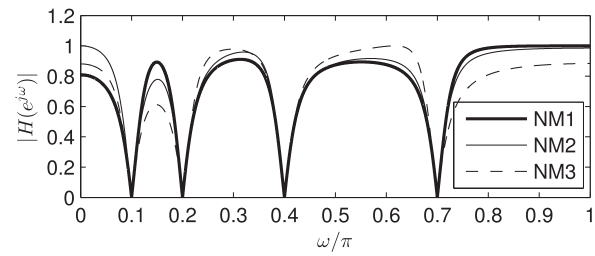
Figure 4 shows the convergence performances of the different NM-methods, whose update parameters are listed in Table 2. For different initializations (including random and specified), the convergent curves are obtained by averaging 100 testing results. It is clear that the convergence of the original NM-method (termed by Nelder) is not sensitive to initialization and better than other ones. Hence, the original NM-method with random initialization is adopted in the following experiments.

3.2 Design Performances with Different Weight Functions

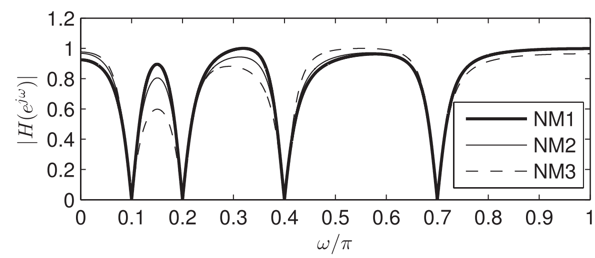
In NM-method based notch filter designs, the empirical weight function is described as (8). Then, we consider the weight functions with other values listed in Table 3. For

Table 3 Values of three weight functions for NM-based notch filter design.

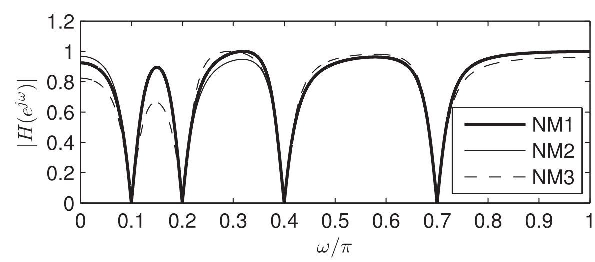
	ω in stop-band	ω in pass-band
NM1	$W_1(\omega) = 2$	$W_1(\omega) = 1$
NM2	$W_2(\omega) = 1$	$W_2(\omega) = 1$
NM3	$W_3(\omega) = 0$	$W_3(\omega) = 1$



(a) Amplitude responses at 30-th iteration.



(b) Amplitude responses at 40-th iteration.



(c) Amplitude responses at 50-th iteration.

Fig. 5 Amplitude responses in iteration procedure of NM-methods with different weight functions.

design specifications shown in Table 1, the amplitude responses of the designed notch filters are shown in Fig. 5, at 30-th, 40-th and 50-th iterations respectively, where NM1, NM2 and NM3 represent three different weight functions shown in Table 3.

In Fig. 5, generally speaking, the design notch filter from NM1 possesses the best amplitude responses, especially between 0.1π and 0.2π. If the iteration numbers are large enough (more than 1000), the amplitude responses from different weight functions approximate identical, but NM1 obtains better amplitude responses early in the iteration procedures. Fig. 5 validates the effectiveness of weight function $W_1(\omega)$, which is same as $W(\omega)$ in (8).

3.3 Performance Comparisons with Classical Method

To illustrate the superiority of the proposed NM-method based notch filter design technique, the performance com-

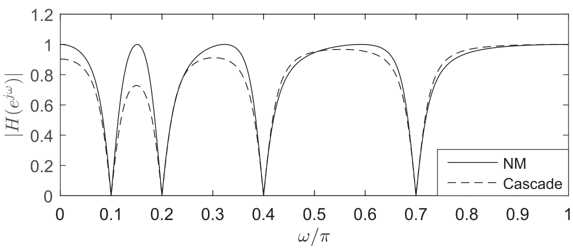


Fig. 6 Amplitude responses of cascade method and NM-method.

Table 4 Specifications for harmonic suppression of ECG signal.

i	1	2	3
ω_i	0.2778π	0.5556π	0.8333π
$\Delta\omega_i$	0.1π	0.1π	0.1π
r_i	0.8524	0.8524	0.8524

parisons are carried out between the proposed method and the cascade one. The design specifications are set as the same in Table 1. The proposed design is implemented via Algorithm 1 and the cascade one is accomplished by (2), through setting $\hat{\omega}_i = \omega_i$, for $i = 1, 2, \cdots, N$. The normalized amplitude responses are shown in Fig. 6.

Although the cascade method is easily implemented without iteration, it is very difficult to get the satisfactory amplitude response under the strict conditions of narrow gap between notch frequencies, for example, $0.1\pi = \omega_1 < \omega < \omega_2 = 0.2\pi$ in Fig. 6. However, for the proposed NM-method based design technique, the amplitude response is greatly improved even though two adjacent notch frequencies are relatively closed.

3.4 Application Case I: Harmonic Interference Suppression

To demonstrate the practicability of the proposed notch filter design technique, we apply it into harmonic interference suppression of electrocardiogram (ECG). The crude ECG signal is sampled from

$$x(t) = x_o(t) + \sum_{i=1}^{3} \sin(2\pi f_i \cdot t) \tag{9}$$

where $x_o(t)$ is the real ECG signal, $f_1 = 50$ Hz, $f_2 = 100$ Hz, and $f_3 = 150$ Hz are the harmonic interference frequencies. The sampling frequency F_s is set as 360 Hz.

To suppress the harmonic interferences in $x(t)$, the design specifications of the desired notch filter are provided in Table 4, where the digital notch frequencies are calculated via $\omega_i = 2\pi f_i/F_s$ $(i = 1, 2, 3)$. The amplitude response of the designed notch filter is shown in Fig. 7. The amplitude response from the proposed design is better than that from the cascade one. Fig. 8 shows the crude ECG signal and the filtered one. It is very clear that the harmonic interferences in the crude ECG signal is effectively suppressed by the designed notch filter.

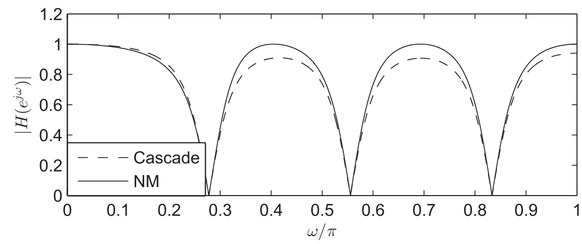
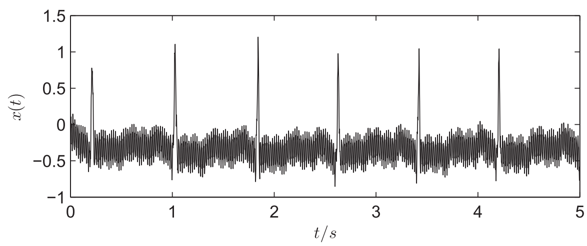
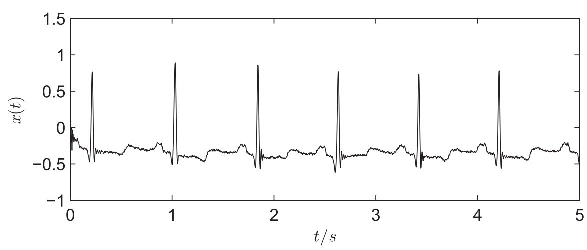


Fig. 7 Amplitude responses of designed notch filters.

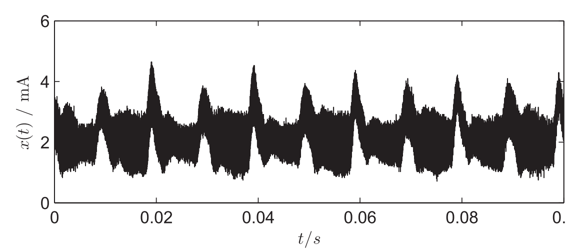


(a) The crude ECG signal.

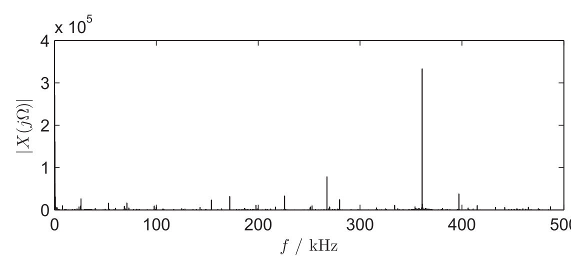


(b) The filtered ECG signal.

Fig. 8 The crude signal and the filtered signals.



(a) Waveform of original corona current.



(b) Spectrum of original corona current.

Fig. 9 The waveform and spectrum of the original corona current.

3.5 Application Case II: Corona Current Denoising

The corona current is usually measured to estimate the corona performance of high-voltage direct current (HVDC)

Table 5 Design specifications for corona current denoising.

i	1	2	3	4	5
f_i	172 kHz	226 kHz	267.4 kHz	361 kHz	397 kHz
Δf_i	9 kHz	9 kHz	9 kHz	9 kHz	9 kHz
ω_i	0.344π	0.452π	0.5348π	0.722π	0.794π
$\Delta\omega_i$	0.016π	0.016π	0.016π	0.016π	0.016π
r_i	0.975	0.975	0.975	0.975	0.975

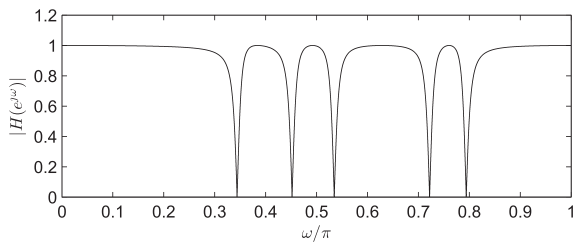
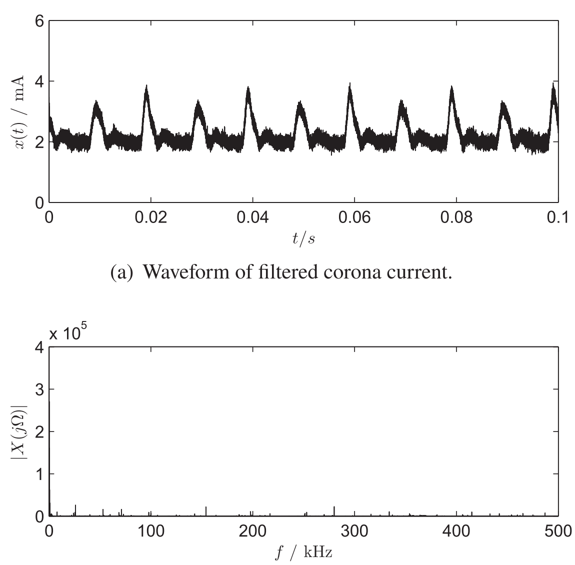


Fig. 10 Amplitude response of designed notch filter.



(a) Waveform of filtered corona current.

(b) Spectrum of filtered corona current.

Fig. 11 The waveform and spectrum of the filtered corona current.

transmission lines. It is affected by a variety of factors, especially amplitude modulation (AM) broadcasts and other radio communications. Under the sampling frequency $F_s = 1$ M Hz, the original corona current and corresponding spectrum are shown in Fig. 9. We can see that there are five notable interference frequencies (from carriers of AM broadcasts) in the corona current.

To suppress those narrow-band interferences (carrier components), the design specifications of the desired notch filter are given in Table 5. The amplitude response of the designed notch filter is shown in Fig. 10. The filtered corona current and its spectrum are shown in Fig. 11. It is clear that the significant narrow-band components in corona current are eliminated effectively.

From above experiments and applications, the effectiveness and practicability of the proposed notch filter design technique have been fully verified.

4. Conclusion

This paper presents a multiple notch filter design technique based on the Nelder-Mead simplex method. The objective function is optimized by minimizing the weighted amplitude response error. The amplitude response is improved by the introduced weight function. Due to the generality of the Nelder-Mead simplex method, the proposed optimization design technique has a simplified structure. Under the same design specifications, the original Nelder-Mead method performs better than other ones. Moreover, the design results demonstrate the superiority of the proposed design technique. Several applications illuminate the effectiveness and practicability of the presented approach.

Future work will pay more attention to reduce computation and improving adaptivity of the designed infinite-impulse-response multiple notch filter.

Acknowledgement

The research is supported by National Natural Science Foundation of China (51207005) and Beijing Natural Science Foundation (3163038).

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